

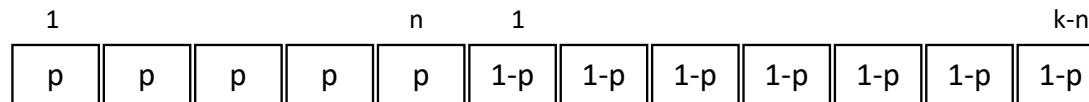
# Binomial distribution

Consider k measurement slots, the probability to detect a particle in a slot is p

k = number of independent slots

p = probability for one particle to be detected in one measurement slot

Mean number of detected particles :  $\langle n \rangle = k \cdot p$



Probability to have n particles detected in the n first slots and no particle detected in the k-n final slots:

$$p^n \cdot (1-p)^{k-n}$$

Probability to have n particles detected in the k slots (independent of the order):  
(combination of n particles in k slots)

→ Binomial distribution

$$B(n) = \binom{k}{n} p^n \cdot (1-p)^{k-n} = \frac{k \cdot (k-1) \dots (k-n+1)}{n!} p^n \cdot (1-p)^{k-n}$$

# Poisson distribution

We use the limits:  
(Poisson distribution)

$$P(n) = \lim_{\substack{k \rightarrow \infty \\ p \rightarrow \langle n \rangle / k}} B(n)$$

- The number k of particles tends to infinity
- The probability p tends to zero
- The mean number detected is k.p

$$P(n) = \lim_{\substack{k \rightarrow \infty \\ p \rightarrow \langle n \rangle / k}} \frac{k \cdot (k-1) \dots (k-n+1)}{n!} p^n \cdot (1-p)^{k-n} = \lim_{\substack{k \rightarrow \infty \\ p \rightarrow \langle n \rangle / k}} \frac{1}{n!} \cdot \frac{k}{k} \frac{(k-1)}{k} \dots \frac{(k-n+1)}{k} (kp)^n \cdot (1-p)^k \cdot \frac{1}{(1-p)^n}$$

$$P(n) = \lim_{\substack{k \rightarrow \infty \\ p \rightarrow \langle n \rangle / k}} \frac{1}{n!} \cdot \langle n \rangle^n \cdot \left[ (1-p)^{1/p} \right]^{\langle n \rangle} = \frac{1}{n!} \cdot \langle n \rangle^n \cdot e^{-\langle n \rangle}$$

Poisson distribution

# Poisson distribution

Mean value

$$\sum_{n=0}^{\infty} n \cdot P(n) = \sum_{n=0}^{\infty} n \cdot \frac{1}{n!} \cdot \langle n \rangle^n \cdot e^{-\langle n \rangle} = \langle n \rangle \cdot e^{-\langle n \rangle} \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \langle n \rangle^{n-1} = \langle n \rangle$$

Quadratic moment

$$\langle n^2 \rangle = \sum_{n=0}^{\infty} n^2 \cdot P(n) = \sum_{n=0}^{\infty} n^2 \cdot \frac{1}{n!} \cdot \langle n \rangle^n \cdot e^{-\langle n \rangle} = \sum_{n=1}^{\infty} \frac{(n-1)+1}{(n-1)!} \cdot \langle n \rangle^n \cdot e^{-\langle n \rangle}$$

$$\langle n^2 \rangle = \langle n \rangle^2 e^{-\langle n \rangle} \sum_{n=2}^{\infty} \frac{1}{(n-2)!} \cdot \langle n \rangle^{n-2} + \langle n \rangle e^{-\langle n \rangle} \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \cdot \langle n \rangle^{n-1} = \langle n \rangle^2 + \langle n \rangle$$

Variance

$$\sigma^2 \equiv \langle (n - \langle n \rangle)^2 \rangle = \langle n^2 - 2n\langle n \rangle + \langle n \rangle^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle$$